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Mathematical Construction for Mechanism Synthesis using Motion Generation Function

Royat M. AL Smadi, Zahratul Amani Zakaria, Elissa Nadia Madi, Bilal M Oraik

***Abstract:** Looking into today's mechanism synthesis we see that this well-developed field has been studied from many perspectives such as; transmission angle, working envelop, Grashof criteria and many others. Also, many mathematical representations were used to solve the problems in this field such as vectorial methods, exponent methods dimensionless analysis, genetic algorithm, artificial intelligence. This paper will be presenting new mathematical approach to solve the mechanism synthesis using motion generation function. This proposed method will study how far the proposed mathematical model (i.e. matrix inversion) will ensure the mechanism to go through prescribed poses or try to approximate them with tolerable error.*

***Keywords:** Mechanism Synthesis, Motion Generation, Prescribe Points, Matrix Approach.*

I. INTRODUCTION

Mechanism synthesis describes the motion of connected links moving relative to each other to achieve certain task through prescribed path. Figure 1 shows four bar mechanism with moving points or pivots ($\mathbf{a_1}$ and $\mathbf{b_1}$) and fixed points or pivots ($\mathbf{a_0}$ and $\mathbf{b_0}$). The mechanism links are described as crank ($\mathbf{a_0}$ $\mathbf{a_1}$), coupler ($\mathbf{a_1}$ $\mathbf{b_1}$) and follower ($\mathbf{b_0}$ $\mathbf{b_1}$).). Figure 1 also shows the motion of mechanism poses in terms of coupler points **p**, **q** and **r.** Looking into other researchers' work, we can find extensive amount of work in this field Charles W Wampler et al., [1] used numerical polynomial mixed with elimination of variables in nonlinear optimization for mechanism synthesis Jaideep Badduri et al., [2] and N Nariman-Zadeh et al.,[3] used genetic algorithem to syhtesize the mechanism. Computer aided design has great deal in mechanism synthesis, Yuxuan Tong et al., [4] relies on geometric constraints, Hans-Peter Zhou and Cheung, [11]) used nonlinear optimization techniques in solving the coordinates of four bar mechanism.

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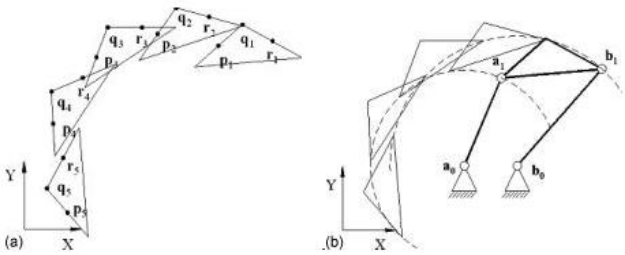

Fig. 1: Proposed mechanism which shows four bar mechanism with prescribed poses [15]

II. NUMERICAL DISPLACEMENT MATRICES BY DIRECT MATRIX INVERSION

This work will present the mathematical formulation and derivation of mechanism synthesis based on Suh and Radcliff [12]. The basic rotation Matrix equation 1 describes the rotation of any vector fixed in a rigid body. The vector is conveniently described in terms of two points fixed in the body, a reference point p at the tail of the vector, and a point of interest q at the head of the vector. For plane rigid body motion (Figure 2), the transformation (i.e. rotation and translation) of points p and q can be written as equation 1

$$\begin{bmatrix} q_x - p_x \\ q_y - p_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} q_{1x} - p_{1x} \\ q_{1y} - p_{1y} \end{bmatrix} \tag{1}$$

Where θ is the rotation of the rigid body with respect to a fixed set of axes. Equation 1 may be written in the compact form

$$(\mathbf{q}-\mathbf{p}) = [R_\theta](\mathbf{q_1}-\mathbf{p_1}) \tag{2}$$

Typically, the original position $\mathbf{p_1}$ and the final position **p** for the reference point are given along with the rotation angle θ. Equation 2 can then be rearranged in a form suitable for calculation of the coordinates of the new position of point **q** when its first position $\mathbf{q_1}$ is specified.

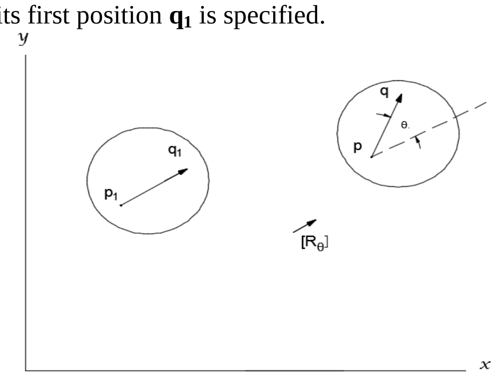

Fig. 2: Plane rigid body displacement.

Solving (2) for **q**, we obtain

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$$\mathbf{q} = [R_\theta](\mathbf{q}_1 - \mathbf{p}_1) + \mathbf{p} \quad (3)$$

Equations (2) and (3) are in a convenient form for carrying out algebraic manipulations for plane motion the rotation matrix is 2×2 matrix.

$$\begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = [D] \begin{bmatrix} \mathbf{q}_1 \\ 1 \end{bmatrix} \quad (4)$$

The 3×3 matrix $[D]$ is the plane displacement matrix. The displacement matrix equation has advantages in repetitive numerical calculations where all matrix elements are defined in terms of the specified displacement of the reference point **p** and the angular displacement of the rigid body. The displacement matrix for pose description is shown in the following equations

$$[D_{1j}] \begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & p_{1y} \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & p_{jy} \\ 1 & 1 & 1 \end{bmatrix} \quad (5)$$

from which

$$[D_{1j}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & p_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & p_{1y} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (6)$$

Note that a constant z coordinate = 1 has been specified for all points. The three displacement equations can be combined in the form. It is sometimes convenient to describe rigid body motion in terms of the known displacement of specified points fixed in the rigid body. For example, the displacement of a plane rigid body could be specified completely by the displacement of three arbitrary non collinear points **p**, **q**, and **r** fixed in the body. Assume that the positions of the points are given as

$P_1(p_1) = P_1(p_{1x}, p_{1y}, 1)$ $\qquad P_2(p_2) = P_2(p_{2x}, p_{2y}, 1)$

$Q_1(q_1) = Q_1(q_{1x}, q_{1y}, 1)$ $\qquad Q_2(q_2) = Q_2(q_{2x}, q_{2y}, 1)$

$R_1(r_1) = R_1(r_{1x}, r_{1y}, 1)$ $\qquad R_2(r_2) = R_2(r_{2x}, r_{2y}, 1)$

Each of the links must satisfy a condition of constant length, where points $\mathbf{a_j}=(a_{jx}, a_{jy})$, $\mathbf{a_0}=(a_{0x}, a_{0y})$, $\mathbf{b_j}=(b_{jx}, b_{jy})$ and $\mathbf{b_0}=(b_{0x}, b_{0y})$ are representative of a typical guiding links $\overline{\mathbf{aa_0}}$ and $\overline{\mathbf{bb_0}}$. These constraint equations become

$$(\mathbf{a_j} - \mathbf{a_0})^T (\mathbf{a_j} - \mathbf{a_0}) = (\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0}) \; j=2,3,\ldots n$$

$$(\mathbf{b_j} - \mathbf{b_0})^T (\mathbf{b_j} - \mathbf{b_0}) = (\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0}) \; j=2,3,\ldots n \quad (7)$$

III. MATHEMATICAL EXPANSION OF CONSTANT LINK EQUATIONS AND ALGORITHM

This work considers solving or synthesizing four bar motion generation by finding the coordinates of moving and fixed pivots. This synthesis will later determine the link lengths that are responsible for moving the mechanism through or approximately from the prescribed poses. The achived coupler points p, q, and r will then be tested agaisnt prescribed poses. The maximum number of presribed poses necessary to solve or synthesize the mechanism follows the equations 8 and 9, where n is the number of phases for the mechanism. Unlike the work done by Bhavi & Math [13]. This work will only focus on single phase mechanism (i.e. the mechanism links can not be dismantled in order to move from one pose to another)

The maximum number of rigid body poses can be described as

$$5+3(n-1) \quad (8)$$

Number of unknowns in the crank or the follower links can be described as

$$2+2n \quad (9)$$

For example, for single phase four bar mechanism (n=1), the maximum number of rigid body poses based on equation 8 is 5, the number of unknows based on equation 9 is 4.

The required unknows are $\mathbf{a_1}=(a_{1x}, a_{1y})$, $\mathbf{a_0}=(a_{0x}, a_{0y})$, $\mathbf{b_1}=(b_{1x}, b_{1y})$ and $\mathbf{b_0}=(b_{0x}, b_{0y})$, so we have two total 8 equations for constant length constraint, with four unknowns a_{1x}, a_{1y}, a_{0x} and a_{0y} for crank and four unknowns b_{1x}, b_1y, b_{0x} and b_{0y} for the follower

$$([D_{12}]\mathbf{a}_1 - \mathbf{a_0})^T ([D_{12}]\mathbf{a}_1 - \mathbf{a_0}) = (\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0}) \quad (10)$$

$$([D_{13}]\mathbf{a}_1 - \mathbf{a_0})^T ([D_{13}]\mathbf{a}_1 - \mathbf{a_0}) = (\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0}) \quad (11)$$

$$([D_{14}]\mathbf{a}_1 - \mathbf{a_0})^T ([D_{14}]\mathbf{a}_1 - \mathbf{a_0}) = (\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0}) \quad (12)$$

$$([D_{15}]\mathbf{a}_1 - \mathbf{a_0})^T ([D_{15}]\mathbf{a}_1 - \mathbf{a_0}) = (\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0}) \quad (13)$$

$$([D_{12}]\mathbf{b}_1 - \mathbf{b_0})^T ([D_{12}]\mathbf{b}_1 - \mathbf{b_0}) = (\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0}) \quad (14)$$

$$([D_{13}]\mathbf{b}_1 - \mathbf{b_0})^T ([D_{13}]\mathbf{b}_1 - \mathbf{b_0}) = (\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0}) \quad (15)$$

$$([D_{14}]\mathbf{b}_1 - \mathbf{b_0})^T ([D_{14}]\mathbf{b}_1 - \mathbf{b_0}) = (\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0}) \quad (16)$$

$$([D_{15}]\mathbf{b}_1 - \mathbf{b_0})^T ([D_{15}]\mathbf{b}_1 - \mathbf{b_0}) = (\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0}) \quad (17)$$

where $[D_{12}]$ and $[D_{13}]$ are known 3×3 displacement matrices that have been precalculated using equation 6 for motion generation. $(\mathbf{a}_1 - \mathbf{a_0})^T (\mathbf{a}_1 - \mathbf{a_0})$ is the Constant length for the crank (R1) and $(\mathbf{b}_1 - \mathbf{b_0})^T (\mathbf{b}_1 - \mathbf{b_0})$ is the Constant length for the crank (R2).

IV. EXAMPLE

The mathematical model described in equations 10 to 17 is heavily dependent on matrix approach to study the motion of the four bar mechanism that move from pose 1 to pose 5 as shown in Figure 5 in Appendix 1. All coordinates are in SI units in (cm), motion generation program can be programed with prescribed values as a_{0x}=0 and b_{0x}=5 and initial guesses as a_{0y}=0, $\mathbf{a_1}$=(3.9392, 0.6946), b_{0y}=0 and $\mathbf{b_1}$=(6.5773,2.5519). The initial guess for contant length of the the crank and the follower are R1=4 and R2=3, repectively. Initial guess is needs for Newton Raphson Technique to solve sequential quadratic programing, this is also done by PS Shiakolas et al., [14], where the author concluded that the synthesis is greatly affected by the quality of initial guess. The prescribed rigid body poses are given in Table I. Using commercially available software like MathCAD, The mathematical model described in equations 10 to 17 can be programed to synthesize the mechanism. The algorithm output will be as the following. $\mathbf{a_0}$=(0,0.2154), $\mathbf{a_1}$=(3.8509, 0.7607), $\mathbf{b_0}$=(5,-0.0107) and $\mathbf{b_1}$=(6.5806,2.5487), crank length (R1)=3.8987 and follower length (R2)=3.0062. The crank angle (θ) corrosponds to 10°, 21°, 36°, 55.9997°, 65.9990°. The synthesized mechanism is shown in Figure 3. The scope of this work is to use motion generation to synthesize four bar mechanism, the synthesized mechanism as shown in Table II goes approximately through the prescribed coupler points in all poses. The yielded scalar error is the absolute difference in any coupler point between the achieved and prescribed poses as shown in Figure 4. The largest error is 0.0321 in coupler point **r** at pose 5.

Table- I: Prescribed rigid body poses for planar four-bar mechanism

	p	q	r
Pose 1	4.3103 , 2.0879	4.6813 , 3.4812	5.2982 , 3.1788
Pose 2	4.4111 , 3.0873	5.3580 , 4.1747	5.7786 , 3.6314
Pose 3	3.9396 , 3.7473	5.0508 , 4.6661	5.3774 , 4.0617
Pose 4	2.9552 , 4.3177	4.2199 , 5.0102	4.4265 , 4.3550
Pose 5	2.3735 , 4.4006	3.7116 , 4.9375	3.8387 , 4.2624

Table- II: Achieved rigid –body poses for planar four-bar mechanism

	p	q	r
Pose 1	4.3103 , 2.0879	4.6813 , 3.4812	5.2982 , 3.1788
Pose 2	4.3950 , 30954	5.3399 , 4.1844	5.7615 , 3.6419
Pose 3	3.9357 , 3.7391	5.0425 , 4.6633	5.3720 , 4.0604
Pose 4	2.9744 , 4.3059	4.2335 , 5.0084	4.4453 , 4.3548
Pose 5	2.4013 , 4.3968	3.7333 , 4.9490	3.8681 , 4.2753

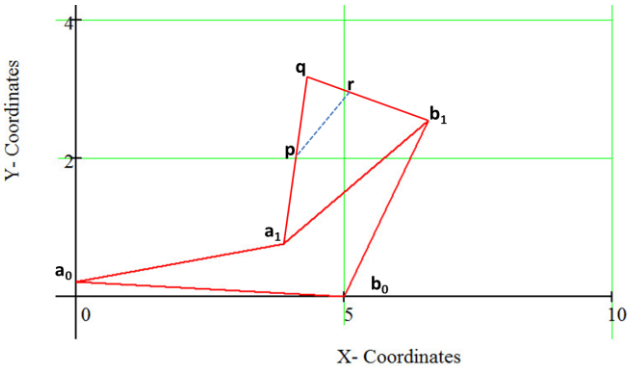

Fig. 3: Four bar mechanism achieved using motion generation generated in MathCAD.

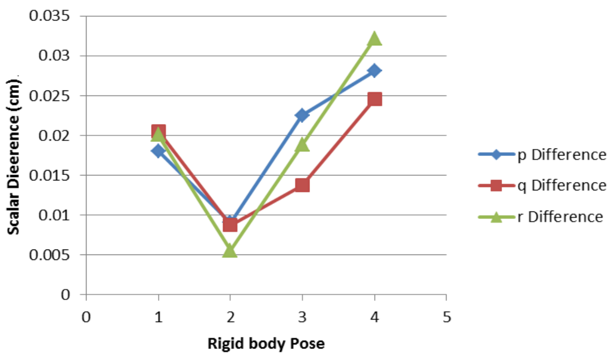

Fig. 4: Scalar difference for coupler points (p,q, and r) between the achieved and prescribed poses

V. DISCUSSION

In motion generation, the required positions of the coupler are given, equation set 10 to 17 depend on the inversion of displacement matrix which is described by the poses of coupler points. The mathematical model will always produce error. However, this error can be also as a constraint in the modelling condition and can be added to the abovementioned equation set as described in the work of Samer Mutawe [16]. Single phase four bar mechanism is discussed in this work. However, multiphase platform can be formulated and programed as well with few modifications depending on equations 8 and 9, for example if there is adjustable mechanism with two phase (n=2), the maximum number of poses reuired is 8 and the maximum number of unknowns is 6.

VI. CONCLUSIONS

Matrix inversion method and motion generation were used to synthesize four bar mechanism. The achieved mechanism motion passes through prescribed poses with very minimum error of 3%. A great deal of attention must be given to the location of coupler point and avoid to put them in one line with fixed ratio, this will result in displacement matrix to have ill condition and cannot be inverted. The mathematical model was programmed using MathCAD and mechanism layout and poses were extracted using CAD software such as Solidworks. The proposed method can be used for any four bar application ranging from low to heavy lifting mechanism.

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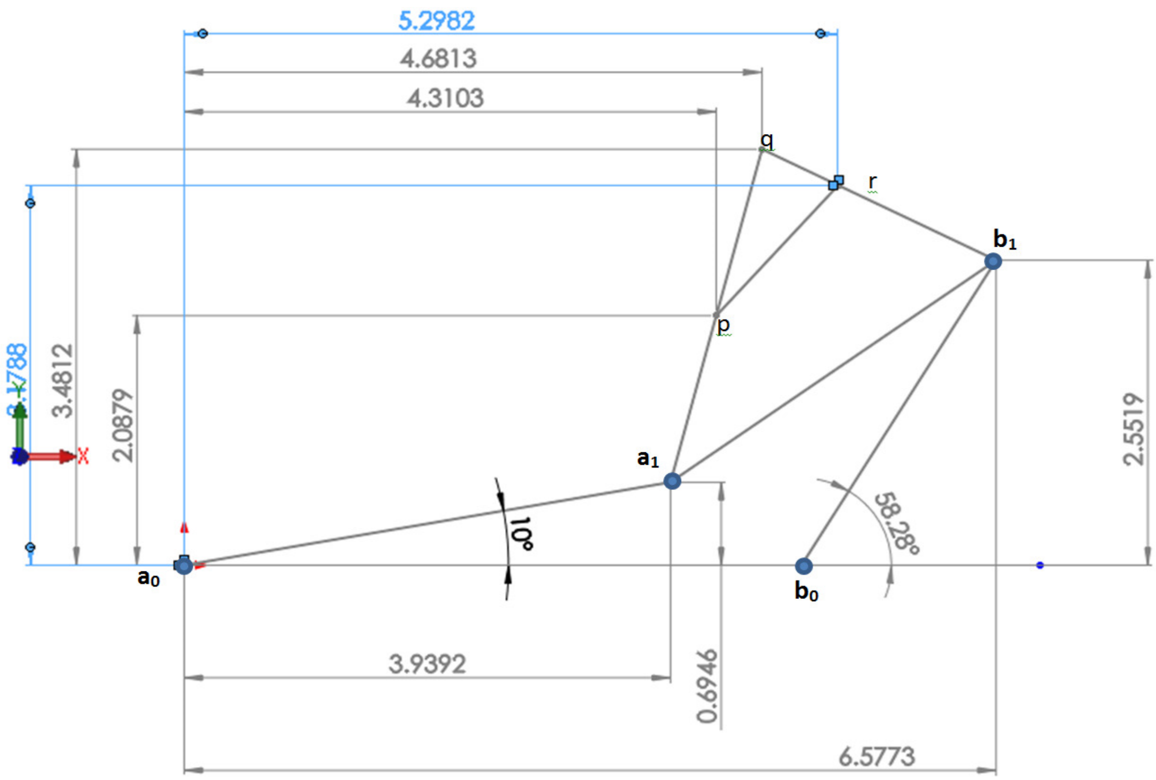
APPENDIX



1st pose

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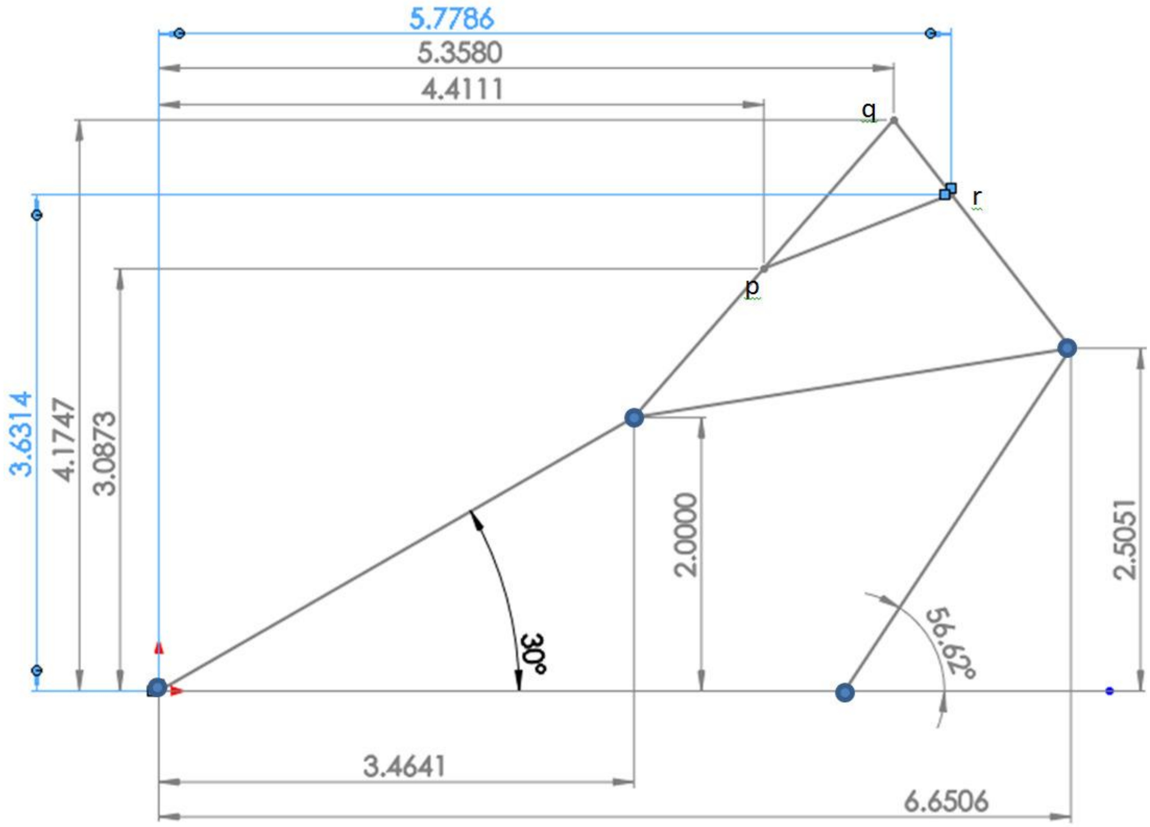
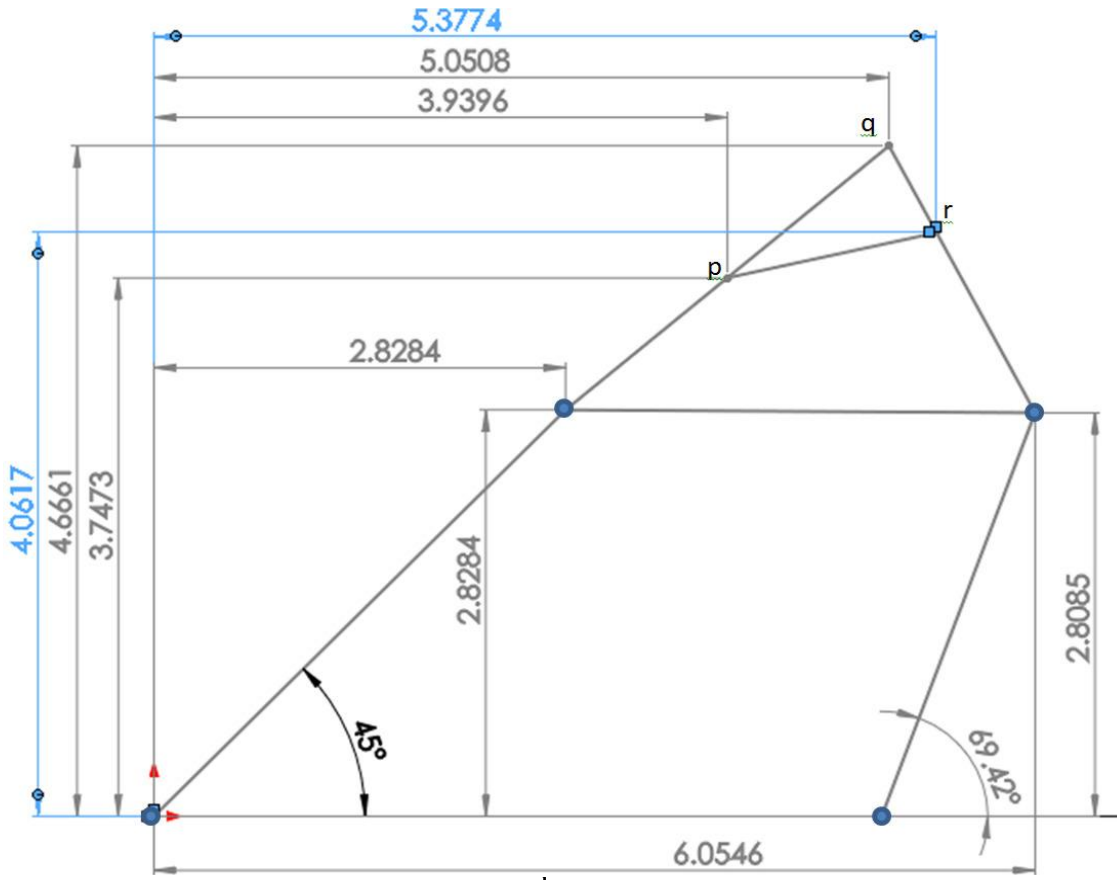
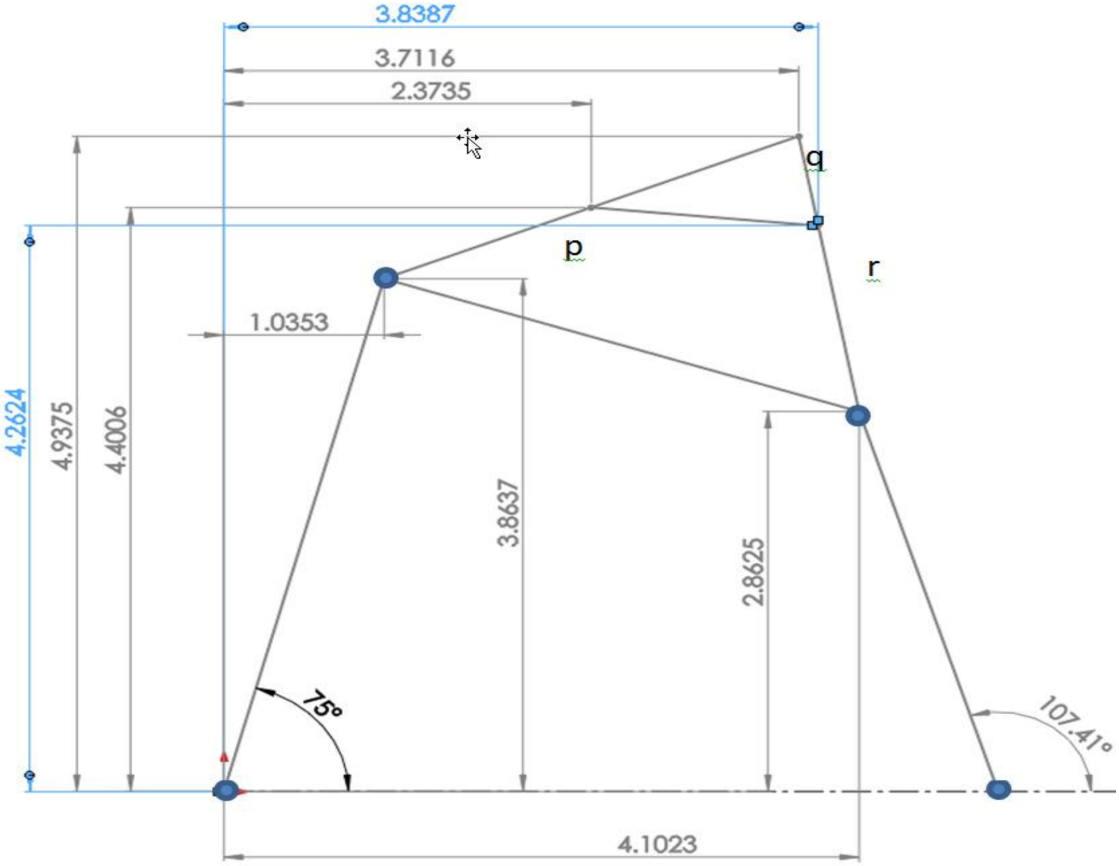


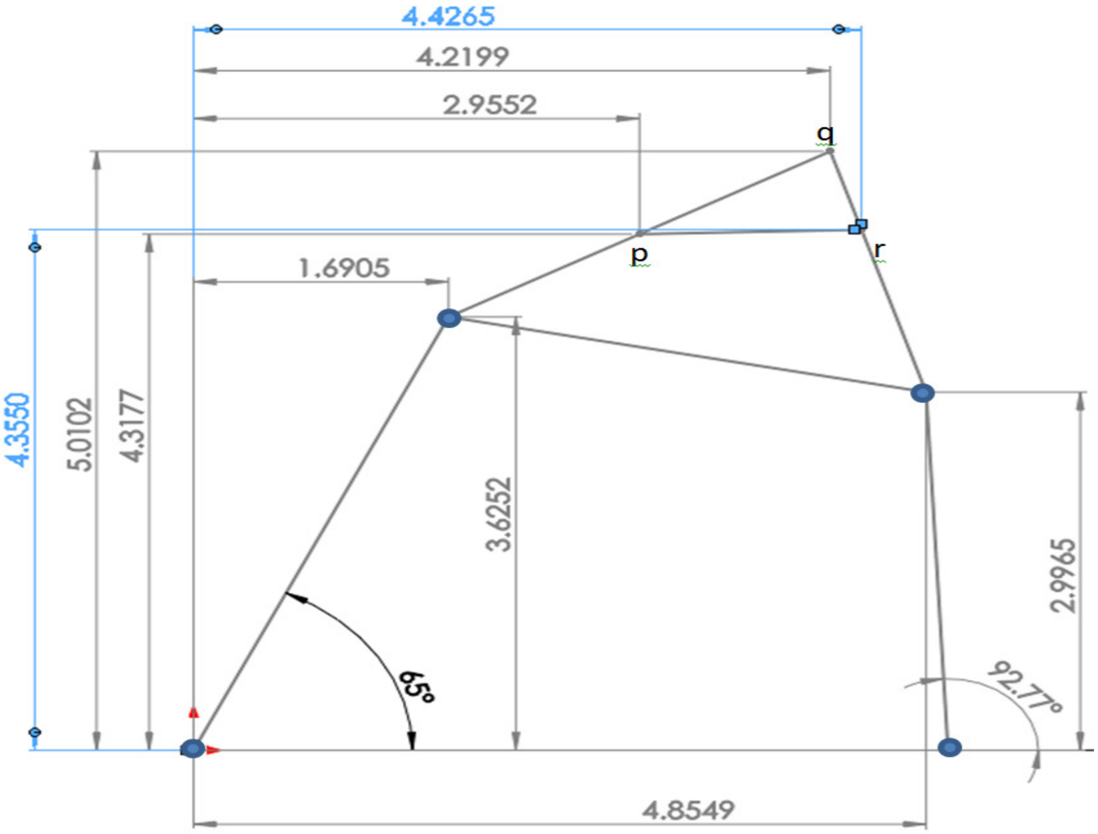
2nd pose



3rd pose



4th pose



5th pose

Fig. 5: four bar mechanism with prescribed poses

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