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Modeling of the Bridge under the Two Vehicles Moving in Opposite Direction with Parametric Study of Bridge Dynamics

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***Abstract:** To ensure the reliability and safety of bridges, one of the main important parameter in design and health assessment is normalized displacement in critical loading condition. It is important to ensure accuracy of determination of normalized displacement in different condition. The aim of this study is to explore the dynamic response of bridge subjected to two vehicles travelling in reverse direction. Moving masses (vehicles) are assumed to start at same time with same speed, then time is delayed for one of the moving mass and then speed of both moving masses is changed. Normalized mid-span response is lesser than that for single moving mass model and highest response for two ways traffic occurs for velocity ratio 0.6. The change of dynamic behavior is remarkable with change in highest normalized mid span response for change in traffic conditions like two way traffic, change in velocity and time of start of vehicles.*

***Keywords:** Bridge-Vehicle Interaction, Two Opposite Moving Masses, Displacement Response, Velocity Variation, Start Time Variation, Normalized Mid-Span, Single Moving Mass, Critical Loading Condition*

List of Symbols

M1	Mass moving on beam (Moving from left to right), Kg
M2	Mass moving on beam (Moving from right to left), Kg
E	Young's Modulus, N/m^2
I	Moment of Inertia, m^4
A	Cross section area m^2
ρ	Density, Kg/m^3
y	Vertical displacement of beam, m
δ	Dirac delta
v_1	Velocity of moving mass M1
v_2	Velocity of moving mass M2
t	Time, seconds
L	length of beam, m
m	Mass of beam, Kg
ω	Natural frequency, rad/s
i and j	Mode Number

I. INTRODUCTION

A huge research is carried out in this area associated with structures subjected to moving load and moving mass to ensure the safety of these structures.

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Many researchers assumed the moving vehicles as moving load to explore the effect of different parameters on the displacement response while many assumed the vehicles as moving mass to consider the effect of its mass inertia which remarkably change the displacement. The mathematical modeling by assuming the vehicles as moving mass is quite complicated due to the resulted fourth order coupled differential equation. Therefore a high contribution in this research area is done to solve these equations and to evaluate with experimental results. Akin and Mofid [1] inserted the orthogonal functions to simplify the governing equation of Euler-Bernoulli beam under the moving mass and solved numerically. The displacements evaluated for the beam of different boundary conditions and also at different locations of the beam. Yang et al. [2] studied the dynamics of railway bridges by modeling the train with two concentrated moving forces with constant distance between them. The effect of span length of the bridge and speed parameters on the dynamics of bridge evaluated and suggested that to avoid the resonant condition the span length of bridge to car length should be equal to 1.5. Ichikawa et al. [3] evaluated the dynamics of the Euler-Bernoulli beam by replacing the one span with multi span continuous beam and solved the governing equation by the central difference method. The response is analyzed with respect to the mass ratio and speed parameter for each span separately and compared. Hilal and Zibdeh [4] determined analytical solution to evaluate the dynamic behavior of beam subjected to a moving force problem. A problem is evaluated for different boundary conditions of the beam. Also the same problem assessed for different types of motion, i.e. accelerating, decelerating and constant speed motion of the mass. The dynamic deflection behavior also evaluated with respect to many parameters. Rao [5] presented the importance of the internal resonance which affects the modal interaction and external resonance by expanding the equation of motion through perturbation method. Identified the instability region of parametric resonance and explained their dependence on moving load to beam mass ratio. Yang et al. [6] modeled a vehicle as oscillator moving over the tight string. A complete coupled system is divided first into distributed system and coupled system to define the equations by Green function for each system and later these functions are combined for the complete system response.

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Wu et al. [7] used the finite element method to find the deflection response of the beam under the moving mass and then solved the mobile crane problem using the same method. Yawari et al [8]s took the rigid bars and resisting joints to form a continuous beam and considered the bending and the shearing effect separately. A governing equation is formulated from Lagrange's equation in matrix form and deflection is determined numerically for the beam with different boundary conditions. Hilal [9] presented the evaluation of dynamic vibration of beam subjected to the random concentrated moving force. This dynamic analysis carried out for the beam with various boundary conditions and for various motion of moving force i.e. accelerating, decelerating and uniform. Pesterev et al. [10] found out a unique function which can easily find the maximum displacement of the beam subjected to the moving force for the given boundary conditions and velocity. Bilello et al. [11] build an experimental set-up by establishing the relation in prototype and small scale bridge with the help of scale factors. The experimental results are evaluated with theoretical results which showed the good agreement in both. Mehri et a.l [12] developed a Green function depending on the boundary conditions of the Euler-Bernoulli beam. The governing equation solved by using this Green function and evaluated the transverse deflection with speed parameters for various boundary conditions. Dehestani [13] introduced the two parameters critical influential speed and coefficient of inertia effect to study the effect of velocity and mass on the dynamic characteristics of the beam. Balut and Kelesoglu [14] presented the two methods Adomian decomposition method and Homotopy perturbation method to find the deflection of beam subjected to moving mass excitation. Ouyang [15] gave a brief review over this area, including the mathematical modeling and focused the importance of investigation of critical velocity. Application of perturbation method explained to solve the equation of motion and also presented the various types of problems which come under this category. Animesh Chatterjee and Tanuja Vaidya [16] used the novel perturbation-based method to present the dynamic effect of the moving mass on the structures like bridge. To present the qualitative insight into the dependence of critical response behaviors on the variable parameters, an analytical expression is obtained. The beam subjected to the moving mass under harmonic and earthquake support excitation was considered by Zarfam et al. [17] to explore its effectiveness in the structural design of bridge located in seismic areas through the study of its response spectrum. This study is carried out for three types of loading system which includes single moving mass, two opposite moving mass and two masses moving with the fixed distance 0.5L between them. The system frequency to the beam frequency ratio with respect to the duration of the time duration required to travel the beam has been explored for the various mass ratio of the moving mass to the beam mass. It was found that the frequency variations in the single moving mass and two opposite moving mass loading system are close to each other for higher modes, while the frequency variations for the two masses moving in same direction with 0.5L distance between them are completely different and less than the other two types of loading. Brady et al. [18] worked out on the dynamic amplification factor for single truck event as well as multiple trucks travelling over the bridge. Finite element model results are evaluated by performing the experiments in Slovenia for single and two trucks running over the bridge. For single truck, the peaks with higher value changes with change in velocities. For two trucks running in opposite direction, the highest peak occurs with 1.11 dynamic amplification factors for the velocity 70 km/hr. For that particular bridge, generally dynamic amplification factor for two trucks travelling in opposite direction with same speed and start at the same time is less compared to that for single truck. Arturo González et al. [19] developed a numerical model to incorporate the single and multiple point loads on the two lane bridge. The effect of the speed and distance between the vehicles prominently studied. The study estimated the critical speeds as a function of natural frequency of bridge and also used to identify the meeting point of the vehicles when the vehicles are travelling in opposite direction on the bridge. Hongye Gou et al. [20] presented experimental and numerical study of moving trains over the long span railway bridge. The dynamic interaction for single train and two opposite moving trains with bridge are estimated for Yexihe Bridge, which is high-speed, longest continuous-girder Railway Bridge in China. The impact factors obtained for two opposite moving trains are lower compare to that for single moving train. This study carried out for various speed and found the critical speed at which the deflection will be maximum. A very few researchers studied the bridge-vehicle interaction problem by considering the two opposite moving mass on the beam. The most of the evidences presented are based experimentation while mathematical modeling and finding the response with numerical method is less evident. Also the effects of time lag of one of the vehicle and if both vehicles travel with different velocity on the dynamic behavior of the bridge is not presented. In this paper, one of the approaches is to determine behavior of the responses of beam under the two moving masses running in opposite directions (traffic flow in opposite directions) with change in velocity of moving masses. Therefore the mathematical model is developed for the bridge under the two moving masses travelling in opposite directions and the algorithm is written in MATLAB to determine the dynamic deflection behavior of the bridge. This deflection behavior is evaluated with speed of vehicles as it is one of the major parameters which affect the deflection. The same mathematical model is then used for another approach to perceive how the entering time of one of the vehicle relative to that of other vehicle affects the vibration of the beam. Also the same mathematical model then varied for the last approach, the vehicles with different speed travel over the bridge in the reverse direction are studied in low speed range and in high speed range. All these approaches suggested that the bridge subjected to two way traffic shows the normalised mid-span response with lesser value compare to that for the single moving mass.

The dynamic response of the single moving mass model is evaluated with the results presented in the earlier literature. The frequency variations for the two way traffic modeling are evaluated with the results presented in the Zarfam et al. [17]. When both the masses are travelling with same speed and started at same time from both end, there is no change in the profile of normalized response curve. While the change in profile of responses can be observed for change in velocity, time of start for one of the mass.

II. DEVELOPMENT OF MATHEMATICAL MODEL AND ALGORITHM

A bridge considered as Euler-Bernoulli simply supported beam with ideal boundary conditions which is subjected to the two vehicles moving in the opposite direction from the two ends of beam is shown in Fig. (1). The following assumptions have been made

i) Vehicle is assumed as a single mass.
ii) Mass M1 is travelling from left to right direction.
iii) Mass M2 is travelling from right to left direction.
iv) Both the masses assumed to start travelling at the same time from both the ends
v) Both masses assumed moving with same constant speed ($v_1=v2$).
vi) The beam is assumed to have a symmetric cross section with constant mass inertia.

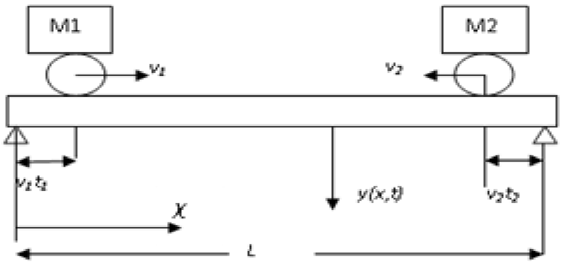

Fig. 1 A simply supported beam under the excitation of two opposite moving masses

It is assumed in the Fig. (1) that a single mass (M1) traversing over the Euler-Bernoulli beam from left to right direction with constant velocity, then the governing equation of motion of beam considering the forces due to weight and inertia of vehicle (M1) can be written as (Ouyang, H. [15]),

$$\begin{aligned} EIy^{IV}(x,t) + \rho A\ddot{y}(x,t) &= M_1 * g * \delta(x - vt) \\ &- M_1[\ddot{y}(x,t) + 2 * v * y'(x,t) * \dot{y}(x,t) \\ &+ v^2 * y''(x,t)]\delta(x - vt). \end{aligned} \tag{1}$$

Now, it is assumed that a single mass (M2) traversing over the Euler-Bernoulli beam from right to left direction with constant velocity, then the governing equation of motion can be written as

$$\begin{aligned} EIy^{IV}(x,t) + \rho A\ddot{y}(x,t) &= M_2 * g * \delta\{x - (L - v_2t)\} \\ &- M_2[\ddot{y}(x,t) + 2 * v_2 * y'(x,t) * \dot{y}(x,t) \\ &+ v_2^2 * y''(x,t)]\delta\{x - (L - v_2t)\}. \end{aligned} \tag{2}$$

And the response of beam subjected to a single moving mass according to Eq.(1) and Eq. (2) is obtained by numerical integration method.

Considering excitation forces because of weight and mass inertia of the two opposite moving masses, a governing equation can be written as

$$\begin{aligned} EIy^{IV}(x,t) + \rho A\ddot{y}(x,t) &= M_1 * g * \delta(x - v_1t) \\ &- M_1[\ddot{y}(x,t) + 2 * v_1 * y'(x,t) * \dot{y}(x,t) + v_1^2 * y''(x,t)]\delta(x - v_1t) \\ &+ M_2 * g * \delta\{x - (L - v_2t)\} \\ &- M_2[\ddot{y}(x,t) + 2 * v_2 * y'(x,t) * \dot{y}(x,t) + v_2^2 * y''(x,t)]\delta\{x - (L - v_2t)\}. \end{aligned} \tag{3}$$

To simplify the equation, expanding the equation by using the principle of superposition

$$y(x,t) = \sum_{j=1}^{N} \psi_j(x) * q_j(t). \tag{4}$$

Whereas, "$q_j(t)$" is the modal response of beam and Mode shape function $i.e.$"$\psi_j(x)$" of healthy simply supported beam is given as

$$\psi_j(x) = \sin\left(\frac{j * pi}{L} * x\right). \tag{5}$$

After using the superposition principle and multiplying with another mode shape function as "$\psi_i(x)$"and an integrating the equation over the domain i.e. (0-L), then applying the orthogonality conditions and using Dirac delta properties, the equation will result as

$$\begin{aligned} &\omega_j^2 q_j(t) + \ddot{q}_j(t) + 2\varepsilon_1 * \sum_{i=1}^{N} \psi_i(v_1t_1) * \psi_j(v_1t_1) * \ddot{q}_i(t) \\ &+ 4\varepsilon_1 v * \sum_{i=1}^{N} \psi_i'(v_1t_1) * \psi_j(v_1t_1) * \dot{q}_i(t) \\ &+ 2\varepsilon_1 v^2 * \sum_{i=1}^{N} \psi_i''(v_1t_1) * \psi_j(v_1t_1) * q_i(t) \\ &+ 2\varepsilon_2 * \sum_{i=1}^{N} \psi_i(L - v_2t_2) * \psi_j(L - v_2t_2) * \ddot{q}_i(t) \\ &+ 4\varepsilon_2 v * \sum_{i=1}^{N} \psi_i'(L - v_2t_2) * \psi_j(L - v_2t_2) * \dot{q}_i(t) \\ &+ 2\varepsilon_2 v^2 * \sum_{i=1}^{N} \psi_i''(L - v_2t_2) * \psi_j(L - v_2t_2) * q_i(t) \\ &= 2\varepsilon_1 g * \psi_j(L - v_2t_2) + 2\varepsilon_2 g * \psi_j(L - v_2t_2). \end{aligned} \tag{6}$$

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$$\begin{aligned}
&\omega_j^2 q_j(t) + \ddot{q}_j(t) \\
&+ 2\varepsilon_1 * \sum_{i=1}^{N} sin(\Omega_{i1}t_1) * sin(\Omega_{j1}t_1) * \ddot{q}_i(t) \\
&+ 4\varepsilon_1 * \sum_{i=1}^{N} \Omega_{i1} * cos(\Omega_{i1}t_1) * sin(\Omega_{j1}t_1) \\
&\qquad * \dot{q}_i(t) \\
&- 2\varepsilon_1 * \sum_{i=1}^{N} (\Omega_{i1})^2 * sin(\Omega_{i1}t_1) \\
&\qquad * sin(\Omega_{j1}t_1) * q_i(t) \\
&+ 2\varepsilon_2 * \sum_{i=1}^{N} sin\left(\frac{\Omega_{i2}L}{v_2} - \Omega_{i2}t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2}L}{v_2} - \Omega_{j2}t_2\right) * \ddot{q}_i(t) \\
&+ 4\varepsilon_2 * \sum_{i=1}^{N} \Omega_{i2} * cos\left(\frac{\Omega_{i2}L}{v_2} - \Omega_{i2}t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2}L}{v_2} - \Omega_{j2}t_2\right) * \dot{q}_i(t) \\
&- 2\varepsilon_2 * \sum_{i=1}^{N} (\Omega_{i2})^2 * sin\left(\frac{\Omega_{i2}L}{v_2} - \Omega_{i2}t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2}L}{v_2} - \Omega_{j2}t_2\right) * q_i(t) \\
&= 2\varepsilon_1 g * sin(\Omega_{j1}t_1) + 2\varepsilon_2 g \\
&\qquad * sin\left(\frac{\Omega_{j2}L}{v_2} - \Omega_{j2}t_2\right). \qquad (7)
\end{aligned}$$

This equation is solved by the Fourth Order Runge Kutta numerical method to show the effect of two masses travelling over the beam on dynamic vibration of the bridge. An algorithm is developed in MATLAB to find the vibration responses when both masses start to travel at the same time. In next model, a vehicle travelling from right to left direction is now delayed by some definite time lag *dk* comparative to an initial time $t_0$of vehicle travelling from left to right direction. The travel duration of both the mass is divided in three phases. A model of the two opposite masses with a time lag for M2 travelling over the bridge is shown below in Fig. (2-4). In the first phase, a vehicle entered from the left end at the time "$t_1 = t_0$"and travel over the bridge. In the next phase, a vehicle entered from the right end at the time "$t_2 = t_k = (t_1 - dk)$" . In the last phase, the only mass M2 will run over the bridge to cover up the time lag after the mass M1 left the bridge.

When $t_1 < dk$

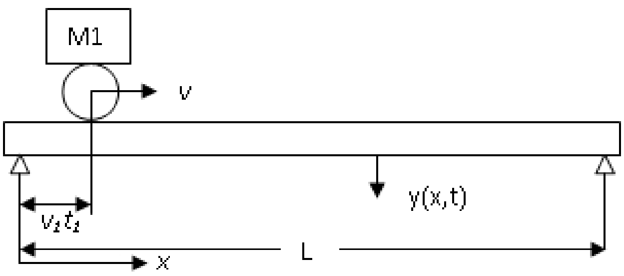

Fig. 2 The first phase when one vehicle start to travel from left direction

When $t_1 >= dk$

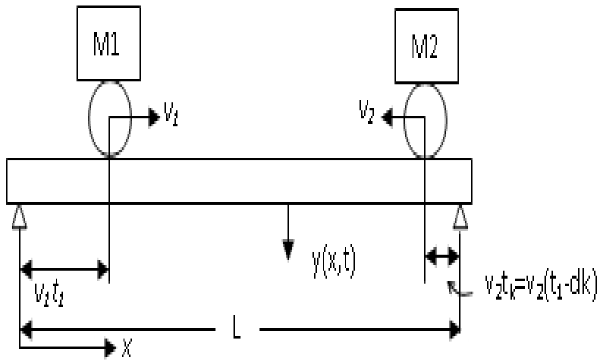

Fig. 3 The second phase when a vehicle entered from the right end of the beam after the given time lag

When $t_1 > L/v$

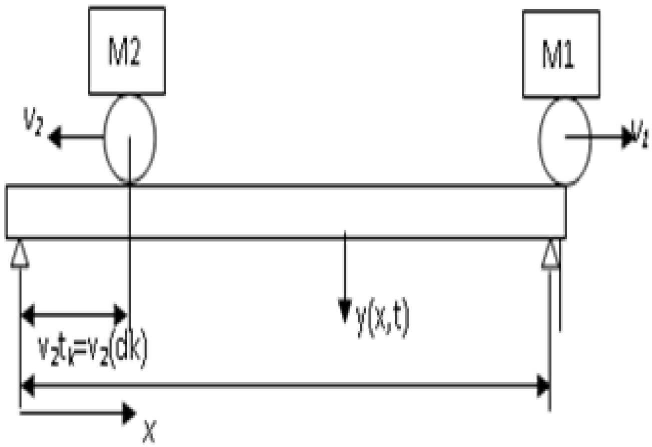

Fig. 4 The last phase when the vehicle M1 will leave the beam and only vehicle M2 will travel

A mathematical model is developed to satisfy these conditions

$$"\alpha_1 = 1 \; for \, 0 \le t_1 \le L/v"$$

$$"\alpha_1 = 0 \; for t_1 > \frac{L}{v}"$$

$$"\alpha_2 = 1 \; for dk \le t_1"$$

$$"\alpha_2 = 0 \; for dk > t_1" \qquad (8)$$

Therefore, inclusion of $\alpha_1 and \alpha_2$ in Eq. (5) results in

$$
\begin{aligned}
&\omega_j^2 q_j(t) + \ddot{q}_j(t) \\
&+ 2 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} sin(\Omega_{i1} t_1) * sin(\Omega_{j1} t_1) \\
&\qquad * \ddot{q}_i(t) \\
&+ 4 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} \Omega_{i1} * cos(\Omega_{i1} t_1) * sin(\Omega_{j1} t_1) \\
&\qquad * \dot{q}_i(t) \\
&- 2 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} (\Omega_{i1})^2 * sin(\Omega_{i1} t_1) \\
&\qquad * sin(\Omega_{j1} t_1) * q_i(t) \\
&+ 2 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} sin\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_k\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_k\right) * \ddot{q}_i(t) \\
&+ 4 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} \Omega_{i2} * cos\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_k\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_k\right) * \dot{q}_i(t) \\
&- 2 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} (\Omega_{i2})^2 * sin\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_k\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_k\right) * q_i(t) \\
&= 2 * \alpha_1 * \varepsilon_1 g * sin(\Omega_{j1} t_1) + 2 * \alpha_2 * \varepsilon_2 g \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_k\right).
\end{aligned}
\tag{9}
$$

$$while, "t_k = t_1 - dk" \tag{10}$$

The Eq. (7) is solved by numerical integration by developing the algorithm in MATLAB to find the response for the two opposite moving vehicles with the same speed but entered on bridge at different time.

Now, mathematical equation is developed when both the masses travelling over the bridge with different velocities.

$$"if\, v_1 < v_2"$$

$$"\alpha_1 = 1 \quad for\ 0 \le t_1 \le L/v_1"$$

$$"\alpha_2 = 1 \quad for\ 0 \le t_1 \le L/v_2"$$

$$"\alpha_2 = 0 \quad for\ t_1 > t_2" \tag{11}$$

Using these conditions in the following equation

$$
\begin{aligned}
&\omega_j^2 q_j(t) + \ddot{q}_j(t) \\
&+ 2 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} sin(\Omega_{i1} t_1) * sin(\Omega_{j1} t_1) * \ddot{q}_i(t) \\
&+ 4 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} \Omega_{i1} * cos(\Omega_{i1} t_1) * sin(\Omega_{j1} t_1) \\
&\qquad * \dot{q}_i(t) \\
&- 2 * \alpha_1 * \varepsilon_1 * \sum_{i=1}^{N} (\Omega_{i1})^2 * sin(\Omega_{i1} t_1) \\
&\qquad * sin(\Omega_{j1} t_1) * q_i(t) \\
&+ 2 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} sin\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_2\right) * \ddot{q}_i(t) \\
&+ 4 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} \Omega_{i2} * cos\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_2\right) * \dot{q}_i(t) \\
&- 2 * \alpha_2 * \varepsilon_2 * \sum_{i=1}^{N} (\Omega_{i2})^2 * sin\left(\frac{\Omega_{i2} L}{v_2} - \Omega_{i2} t_2\right) \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_2\right) * q_i(t) \\
&= 2 * \alpha_1 * \varepsilon_1 g * sin(\Omega_{j1} t_1) + 2 * \alpha_2 * \varepsilon_2 g \\
&\qquad * sin\left(\frac{\Omega_{j2} L}{v_2} - \Omega_{j2} t_2\right).
\end{aligned}
\tag{12}
$$

Using the condition given in Eq. (11), Eq. (12) is solved by Fourth Order Runge Kutta numerical method, which gives the result of the two vehicle models running with different velocities in reverse direction. All the vibration responses show the cumulative effects of three modes as the equations are expanded here up-to three numbers of modes of vibration.

III.RESULTS AND DISCUSSION

A. Two opposite masses: assuming both the vehicle enters the bridge at the same time from opposite ends

A beam with geometric dimensions *as "L=20m, m=312Kg/m" and with material properties as "E=2.06×10*^{11} *N/m*^{2}, and *ρ=7,800 kg/m*^{3}*"* is considered for the complete analysis. While the mass ratio parameter is taken as "$\varepsilon_1 = \varepsilon_2 = 0.1$" and velocity ratio, i.e. VR ranges in between "($\mu = 0.1 - 1$)". The normalized mid-span displacement responses are plotted against the moving mass position for all the cases. The displacement is normalized by dividing the dynamic response of bridge with the static maximum response of the beam. The displacements of the bridge under two moving vehicles in the opposite direction are compared with the displacement of the bridge subjected to single moving mass with mass ratio (ε=0.2).

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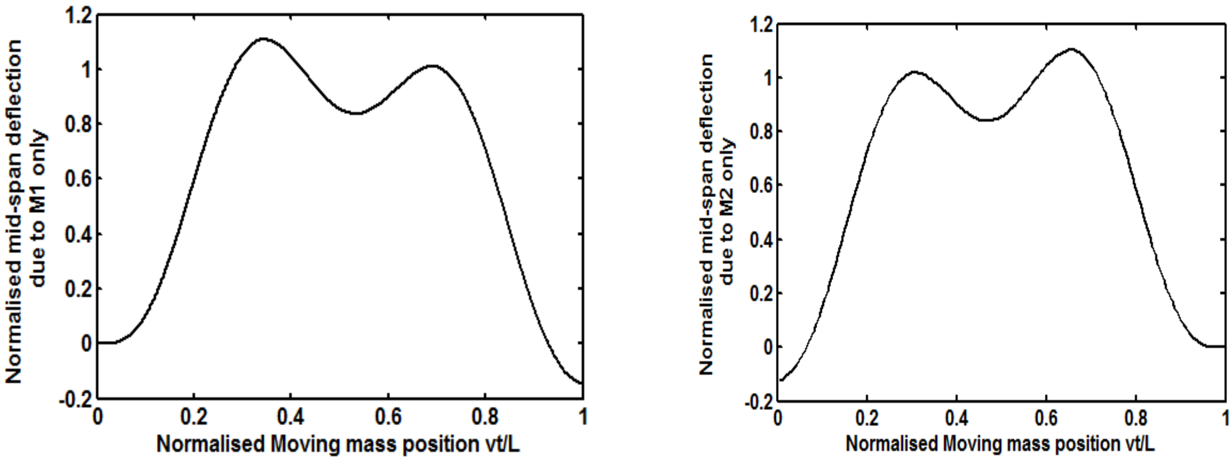
Fig. (5a) is shown for the dynamic deflection of the beam under a single moving mass (M1) travelling from left to right direction with reference to Eq.(1) and Fig.
(5b) is shown for mass M2 traversing from right to left direction with reference to Eq.(2) for the above mentioned geometric dimensions and properties of the beam with mass ratio "$\varepsilon_1 = \varepsilon_2 = 0.1$"and velocity ratio "$\mu = 0.2$". Fig.5a and b shows the effect of direction of travelling of moving mass on the behavior of the displacement of beam. These curves show the change in displacement curve as the direction of travel of mass changes. The positions of both the peaks of normalized mid-span displacement curve are changed. Further these two cases are combined to see the effect of the two masses travelling in opposite direction on the beam.
Here an attempt is made to resemble the modeling of the problem that includes masses running over the beam in opposite direction with the problem of dynamic behavior of bridge subjected to the two way traffic. From the Figs.(6-8), it can be stated that for a lower velocity ratio, the dynamic vibrations of the beams subjected to the excitation of two opposite travelling masses is similar to that of single moving mass on beam. Even there is no change in profile of response curve and location of maxima of response. A considerable difference between normalized mid-span response of single moving mass and two opposite moving mass model can be observed for the speed ratio greater than 0.4 (V/Vcr>0.4) which corresponds to the vehicle speed of 66.67 km/hr. This is due to the fact that at lower velocity, the change in mass of vehicle affects the structural vibration very less compare to that at higher velocities which is depicted effectively in Fig (9). The dynamic normalized mid-span response (NMSR) of the beam for two opposite moving mass model is observed to reduce with increase in velocity compare to that of the single moving mass model which is similar to that estimated by [2, 7, 8]. Specifically the difference between the NMSR of two opposite mass and single mass model is distinctively noticeable from the 30% length of the beam. The speed at which value of NMSR maxima is highest also changed and it is highest at the velocity ratio 0.6 and the value of NMSR maxima reduces further with increase in velocity ratio. The location of response maximum retained same as it is in single moving mass model which can be observed for all velocity ratios. The NMSR for two moving mass case is higher for velocity ratio VR 0.6 while it is 1 for single moving mass case. So, it can be said that the resonance condition where the frequency of two moving masses running over the bridge becomes same as of the bridge for VR 0.6 while VR for single moving mass is 1.
The dynamic displacement curve for single moving mass shows that mid-span displacement peak attains the higher value above the static maximum displacement with the increase in velocity up to critical velocity. It attains almost above the double of the static displacement for the critical velocity. While for two opposite travelling masses, the highest value of dynamic displacement curve is below the double of static maximum displacement that too for velocity ratio 0.6. Here it is important that according to the research theory [1-9] carried based on single moving mass depicts that the mid-span displacement curve attains the highest value at the critical velocity and increase in the velocity beyond the critical velocity of moving mass, the value of peak of displacement curve reduces. But for two opposite moving mass model, the higher value of the mid span displacement attained for 0.6 velocity ratio. It is necessary to avoid the velocity ratio those causes higher displacement of the beam for the prolonged life and safety of the structure.



a) Mass (M1) traversing from left to right **b) Mass (M2) traversing from right to left**

Fig. 5The Response of Beam Under a Single Moving Mass

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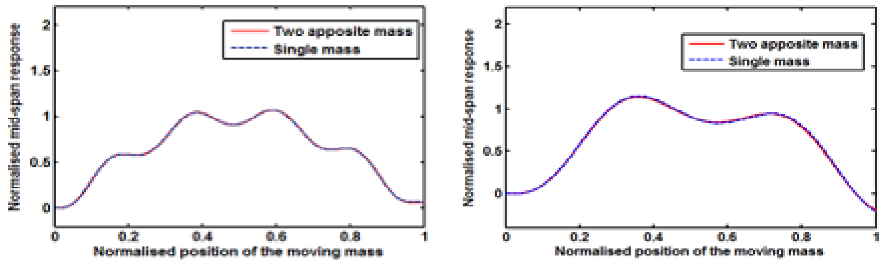
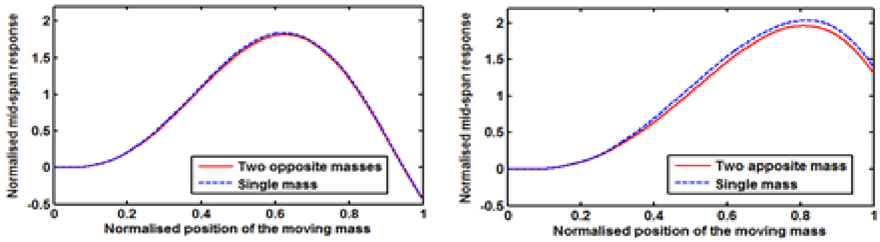


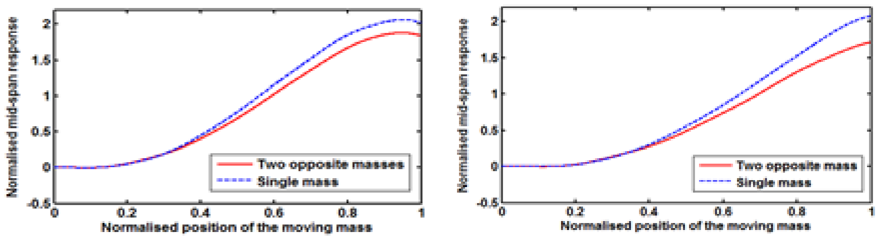
(a) Velocity ratio=0.1 (b) Velocity ratio=0.2

Fig. 6 Mid-span normalized displacement response of beam under two opposite moving masses (a) VR=0.1, (b) VR=0.2



(c) Velocity ratio=0.4 (d) Velocity ratio=0.6

Fig. 7 Mid-span normalized displacement response of beam under two opposite moving masses (a) VR=0.4, (b) VR=0.6



(a) Velocity ratio=0.8 (b) Velocity ratio=1

Fig. 8 Mid-span normalized displacement response of beam under two opposite moving masses (a) VR=0.8, (b) VR=1

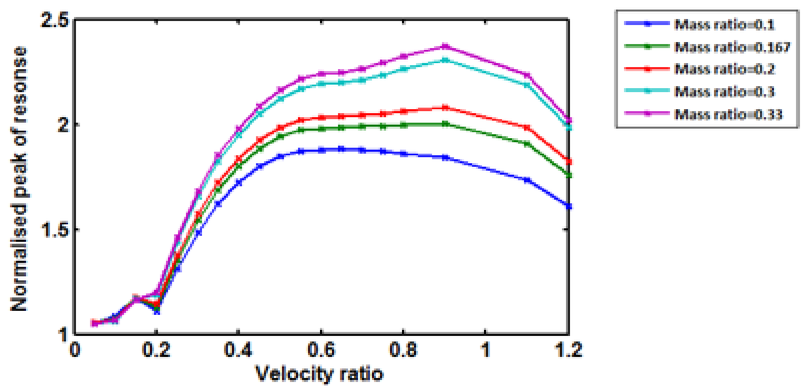


Fig. 9 peak of mid-span response of simply supported beam under single moving mass for velocity ratios (vr) by changing the moving mass to beam ratio (ε)

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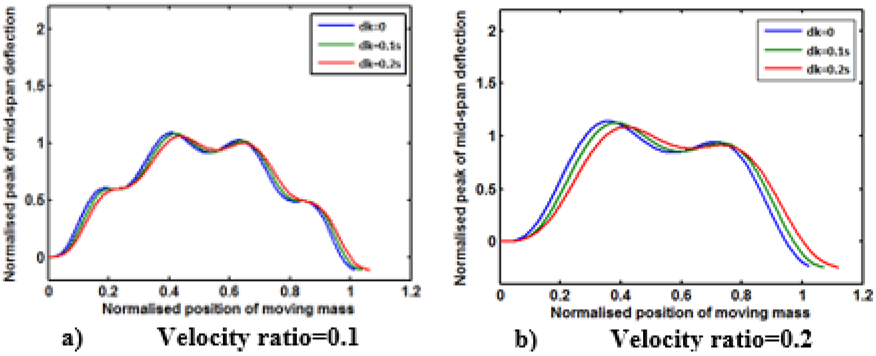


Fig. 10 Mid-span displacement response of the beam under two opposite moving mass excitation with time delay a) VR=0.1, b) VR=0.2

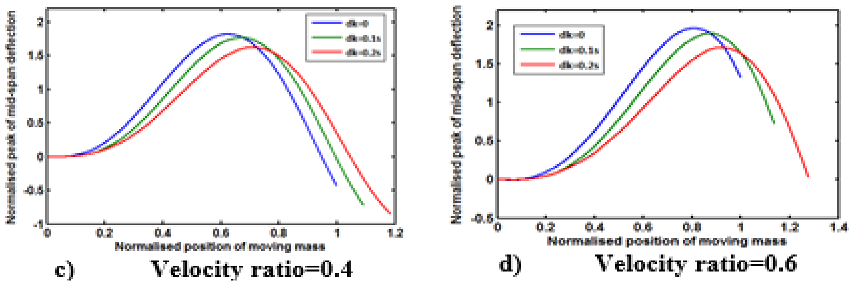


Fig. 11Mid-Span Displacement Response of The Beam Under Two Opposite Moving Mass Excitation With Time Delay

B. Two opposite masses travelling in the opposite direction: with time delay

A mass M2 running over the beam from right to left direction is being delayed with a time lag to see the effect on the dynamic response. The mass M1 and M2 have same weight and velocity. The equation (9) is solved numerically to find the dynamic response with addition of time lag for M2. The time delays provided are 0.1s and 0.2s. The results are then compared with that for zero time lag, i.e. both the masses starts to run over the bridge at the same time.

Even at lower velocity, the effect of delayed entry of the mass M2 can be observed in the Figs. (10-12). A remarkable decrease in the response values are obtained with increase in time lag. Also there is not any change in the profile of response curve. The location of response peak is also shifting toward right with increase in time lag. The reduction in response values with increased time lag is more for moving mass traversing with high velocity. The decrease and shift in NMSR peak shows that the meeting point of two moving masses is shifting away from the mid-span towards the right as in [18]. So, here it can be stated that two vehicles traversing in opposite direction with time lags results in reduced bridge response.

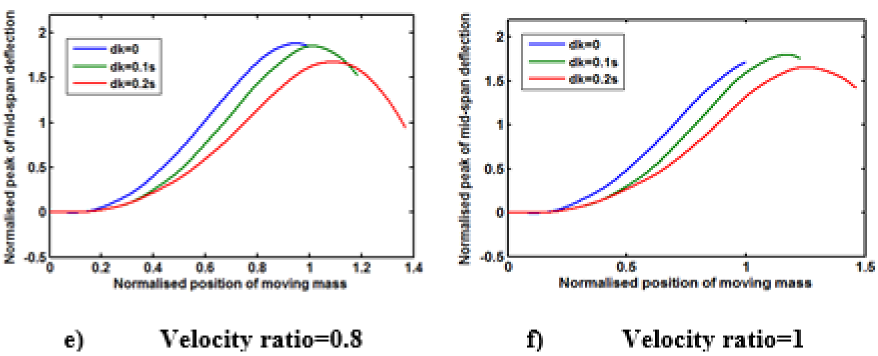


Fig.12 Mid-span displacement response of the beam under two opposite moving mass excitation with time delay

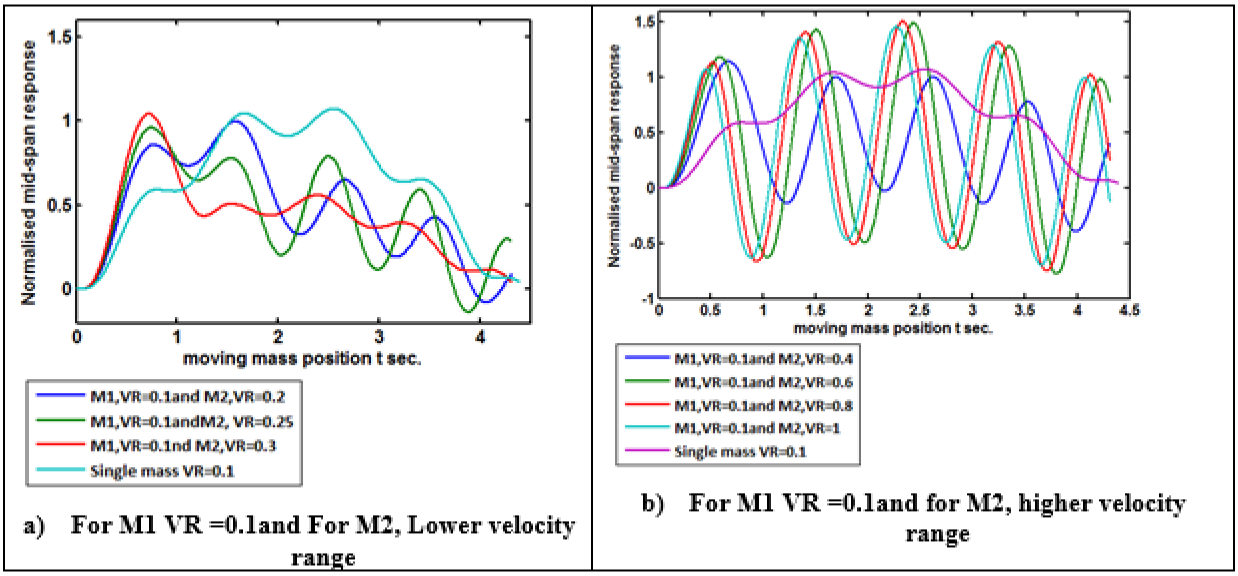


Fig. 13 Mid-span Displacement Response Compared with That for Single Mass

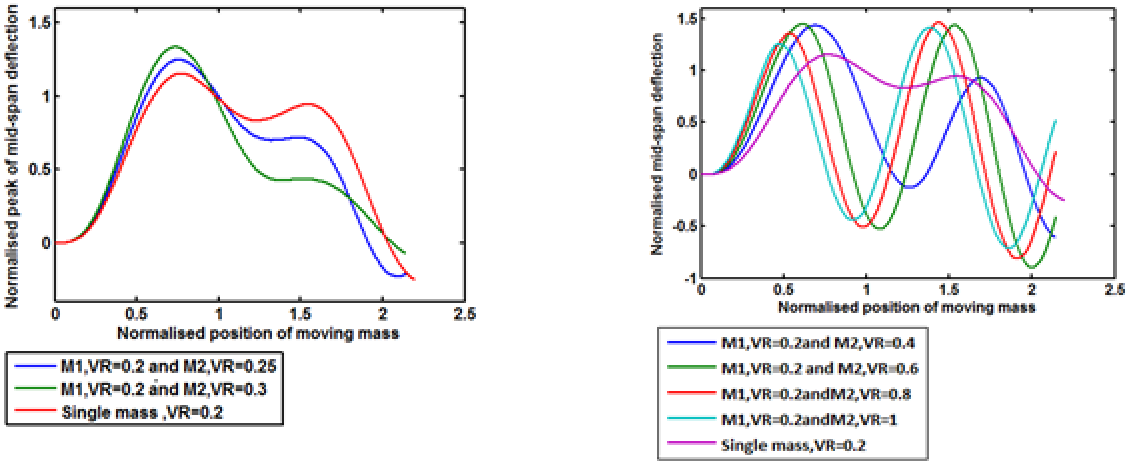
C. Two masses running over the bridge with different velocity

Equation (12) is solved to find the response of the beam under the two moving masses traveling in opposite direction with different velocity. In the previous section, it is observed that at lower velocities, there is no difference in the dynamic response of two mass moving in the reverse direction model and the single moving mass model. While for higher velocities, the mid-span normalized dynamic response of the two moving mass model has lower values compare to that of the single moving mass. The effect of speed of vehicles running over the bridge is higher on the bridge response and therefore the speed of vehicles is now assumed to be different. Again in this model both the masses start to travel at the same time.

Initially, the velocity ratio of the mass M1 is kept 0.1 and the velocity ratio of mass M2 is varied in the range of 0.2-1 keeping the mass of M1 and M2 same. Fig. (13 a,b) shows higher value of NMSR peaks occurs at the time travel of 0.7 second (16% of the total travel time) which is observed up to the velocity ratio 0.4. The increase in velocity ratio of M2 up to 0.25 results in comparative lower value of NMSR peak. Also the change in profile of curve is observed with change in velocity of M2. Number of peaks appeared in the curve are same as for single moving mass and two opposite moving masses with same velocity.

For the mass M1 with VR 0.1 and M2 with VRs 0.6-1, the normalized displacement responses have considerably higher values compare to the NMSR for single moving mass with VR 0.1. The higher peaks are observed to take place near the 2.5 second (57% of the total travel time). The higher value of peak is observed for the velocity ratio 0.6 and 0.8. The numbers of peaks are also observed to be increased. The value of mid-span dynamic response is about 1.5 times of static response even for higher velocities. The Fig. (14 a) shows the response of beam subjected to the excitation of M1having velocity corresponding to velocity ratio 0.2 (V/Vcr=0.2) and M2 with velocity ratio of 0.25 and 0.3. The responses are compared with the response of single mass moving with velocity ratio 0.2. The higher value of NMSR peak again occurs at the time travel of 0.7 seconds (32% of the total travel time) with considerably higher value of NMSR compare to that for single moving mass with VR 0.2. While for M1 with VR 0.2 and M2 with VRs 0.6-1, the higher value of NMSR peaks occurs near the 1.5 second of time travel which is around 68% of total travel time. The value of NMSR for these two moving masses is also observed to be comparatively higher to those for single moving mass with VR 0.2 as shown in Fig. (14 b). The value of mid span dynamic response still has value less than 1.5 times of static response. The numbers of peak are observed to be more for the velocity ratio 0.4-1 than it is for velocity ratios 0.2-0.3. For velocity ratio 0.2- 0.3, only two peaks appear in the curve. The higher value of peak NMSR is again observed for velocity ratio 0.6 and 0.8.

In Fig. (15), the VR of M1 changes to 0.4 and for M2 to 0.6-1, the value of higher peaks of NMSR is decreasing and shifting towards left with increase in VR of M2. A remarkable decrease in the values of NMSR is observed. The similar behavior of NMSR is observed for M1 with VR 0.6 and M2 with VRs 0.8 and 1. Even for higher velocity ratios, the mid-span dynamic response is below the two time of static response, while for single moving mass the mid-span dynamic response is above the two times of static response.



a) For M1, VR=0.2 and For M2, lower velocity range

b) For M1,VR=0.2 and For M2, higher velocity range

Fig. 14 Mid-span displacement of the compared with that for single mass

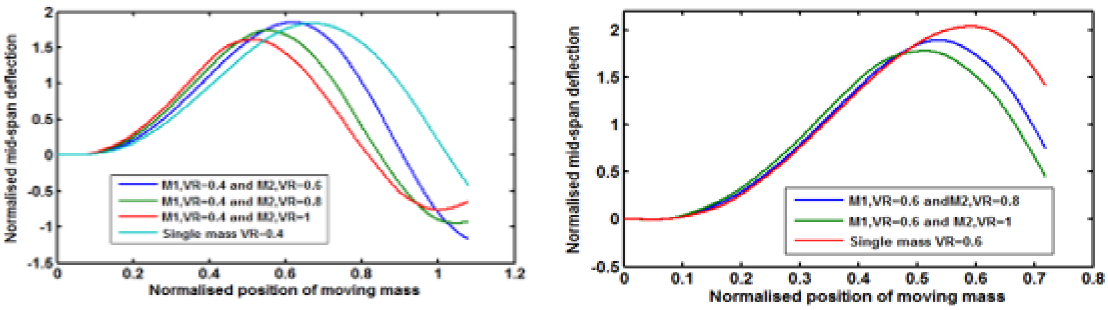


Fig. 15 Mid-span displacement response of the two opposite masses compared with that of single mass for M1 and M2 both in high velocity range

It is noticeable that when M1 and M2 both travel with lower velocities, only one peak attains the higher value. This behavior of dynamic response curve is observed up to velocity ratio 0.3. When M1 travels with low velocity ratio and M2 travels with high velocity ratio that is above 0.4, the value of all peaks is marked to be increased. When M1 and M2 both travel with high value of velocity ratio, there is no change in the profile of displacement curve. Also it is marked that with the increase in velocity of M2, the response peaks shifted towards left.

IV. CONCLUSION

To ensure the accuracy in the design of structures like bridge, the accurate estimation of maximum displacement factor is necessary. The maximum displacement factor is analyzed considering various conditions of traffic. The analysis is carried out to perceive the effect of vehicles running in reverse direction over the dynamic behavior of the bridge. This study is first carried out to make out the difference in response due to single vehicle compare with the two vehicles running in reverse direction keeping mass ratio, velocity and their starting time same. For low velocity ratios 0.1-0.4, there is not any remarkable change in the normalized mid-span response, but normalized mid-span dynamic response NMSR of the beam reduces remarkably for high velocity ratios 0.6-1compare to that for single moving mass. The high value of NMSR for two moving mass is observed for VR 0.6, while for single moving mass high value of NMSR observed for VR 1. It is important to find the velocity ratio, at which the displacement of the bridge will be higher for the prolonged life and safety of these structure. This model is then aided with the delay in the starting time of one of the mass to see the effect on the dynamic response of the beam. The delay in starting time of travel of one of the mass i.e. M2 reduces the NMSR response of the beam to a greater extent. The peaks of response are observed to shift towards the right with increased time lag. This decrement in the response is more with the increment in the time delay. So it can be said that with increased time lag the meeting point of two masses shifted away from the mid-span towards right. While this investigation needs more detailed study to conclude.

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When two masses travel with different velocity, the profile of NMSR changed when M1 and M2 travel with low velocity ratio that is below 0.3. Also the same observed when M1 with low velocity ratio and M2 with high velocity ratio travel over beam. There is no change in the profile of displacement curve, when both masses travel with high velocity ratio. All peaks of response curve shifts towards left with increase in the velocity of M2. When both masses travel with low velocity ratio, mid-span dynamic response has the highest amplitude value near the static response. When M1 travels with low velocity and M2 with high velocity, the mid-span dynamic response reached to approximately 1.5 times of the static response. When both masses travel with high velocity, the mid-span dynamic response attained approximately 1.8 times of the static response. In the broad sense, it is observed that the maximum displacement factor occurs for different speed compare to that one for single moving mass. While with the introduction of time lag, the maximum mid-span displacement response is observed to get reduce with change in its occurring location. When the two masses are travelling with different velocities, the normalized maximum mid-span displacement is between 1and 1.5 for the lower and higher velocity combination. The normalized displacement for higher velocities combination lies between the 1.5 and 2. The maximum normalized displacement is observed for one vehicle with 0.4 and other with 0.6. But still this normalized mid span deflection is lower compare to that it is for single moving mass. Here the bridge is modeled as simply supported beam and the maximum deflection value and curve behavior may vary with the change in boundary conditions of the beam.

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